一般化極値分布、3 母数ワイブル分布の最良かもしれない推定法 KLDMWR-GE

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Introduction

- For the three-parameter Weibull (TPW) and the generalized extreme value (GEV) distributions, the maximum likelihood (ML) method fails for some range of parameters.
- We find that our KLDMWR-GE, which is equivalent to Jiang's modified maximum product of spacings (JMMPS) method, is applicable to these distributions for all ranges of parameters, and in many cases has less bias than the original MPS.
- We know there are many alternatives to ML for TPW and GEV, but, in this presentation we only investigate the relative performance of KLDMWR-CE (ML), SE (MPS) and GE (JMMPS).

Introduction 2

TPW

$$F(x;\nu,\eta,\xi) = 1 - \exp\left[-\left(\frac{x-\nu}{\eta}\right)^{\xi}\right] \quad (1)$$

For ML,

- when $\xi < 1$, MLE does not exist or inconsistent;
- when $1 \geq \xi < 2$, MLE may exist but not asymptotically normal;
- when $2 \leq \xi$, MLE exists and asymptotically normal.

In contrast, for KLDMWR-SE (MPS) and GE (JMMPS),

- when $\xi < 2$, the SE and GE exist but not asymptotically normal.

Introduction 3

GEV

$$G(x; \nu, \eta, \xi) = \exp\left[-\left\{1 + \xi\left(\frac{x-\nu}{\eta}\right)\right\}^{-1/\xi}\right]. \qquad (2)$$

For ML,

- when $\xi < -1$, MLE does not exist or inconsistent;
- when $-1 \geq \xi < 0.5$, MLE may exist but not asymptotically normal;
- when $-0, 5 \leq \xi$, MLE exists and asymptotically normal.

In contrast, for KLDMWR-SE (MPS) and GE (JMMPS),

- when $\xi < -0.5$, the SE and GE exist but not asymptotically normal.

It is known that the TPW is a reparameterization of GEV, two distributions are closely related.

Simulation

- We generated samples using the probability integral transformations.
- For TPW, $\nu_0 = 1$, $\eta_0 = 1$, $\xi_0 = 0.5$: inconsistent ML, $\xi_0 = 1.5$: non-asymptotically normal ML, $\xi_0 = 2.5$: regular ML.
- For GEV, $\nu_0 = 1$, $\eta_0 = 1$, $\xi_0 = -2$: inconsistent ML, $\xi_0 = -0.75$: non-asymptotically normal ML, $\xi_0 = 0.75$: regular ML.
- Sample size $n \in \{20, 50, 100, 200, 500\}$.
- Number of samples: 10000.

MSE, TPW

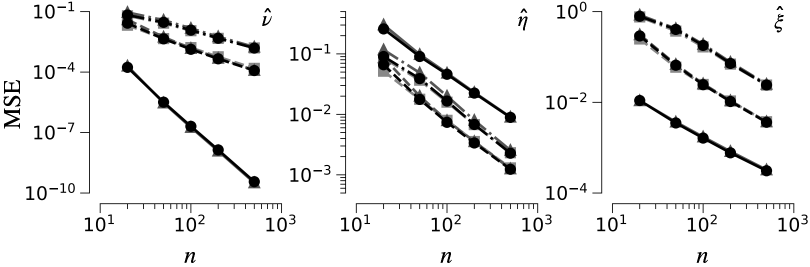

Figure: TPW, MSE vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = 0.75$, Dotted line: $\xi_0 = 1.5$, dot-dash line: $\xi_0 = 2.5$.

Bias, TPW

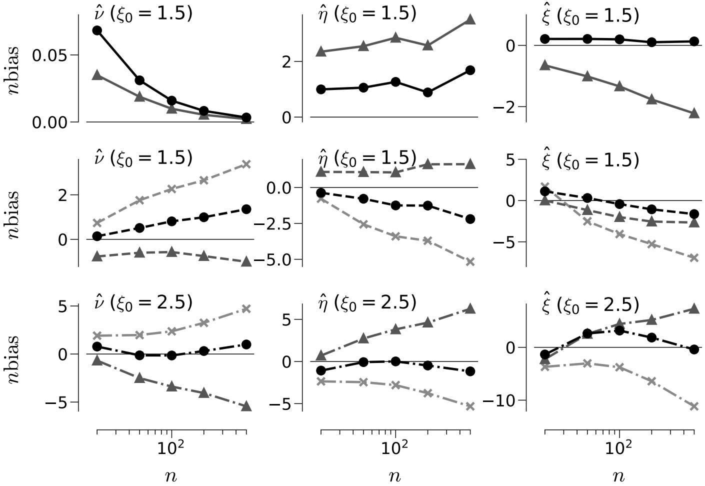

Figure: TPW, $n$bias vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = 0.5$, Dotted line: $\xi_0 = 1.5$, dot-dash line: $\xi_0 = 2.5$.

Bias of return levels at $p = 0.01$, TPW

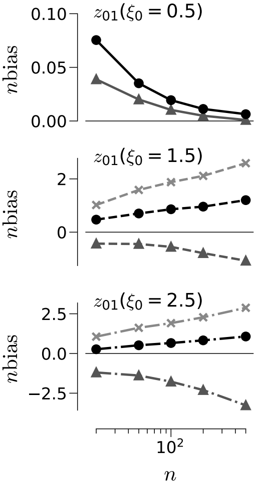

Figure: TPW, $n$bias vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = 0.5$, Dotted line: $\xi_0 = 1.5$, dot-dash line: $\xi_0 = 2.5$.

MSE, GEV

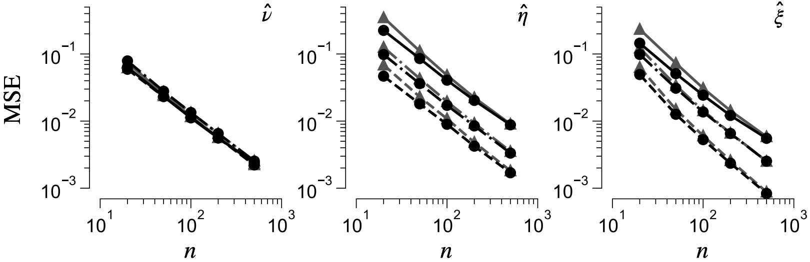

Figure: GEV, MSE vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = -2.0$, Dotted line: $\xi_0 = -0.75$, dot-dash line: $\xi_0 = 0.75$.

Bias, GEV

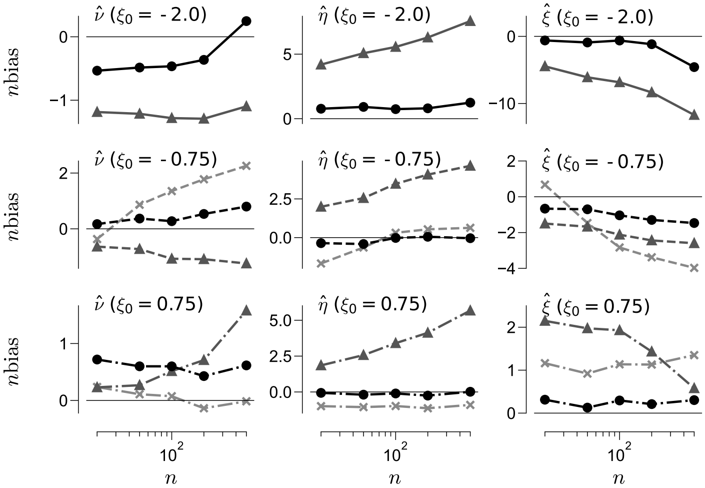

Figure: GEV, $n$bias vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = -2.0$, Dotted line: $\xi_0 = -0.75$, dot-dash line: $\xi_0 = 0.75$.

Bias of return levels at $p = 0.01$, GEV

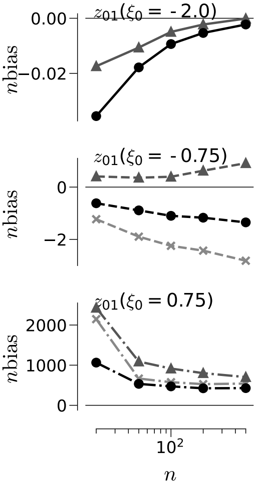

Figure: GEV, $n$bias vs sample size; circle: GE, triangle: SE, cross: MLE; solid line: $\xi_0 = -2.0$, Dotted line: $\xi_0 = -0.75$, dot-dash line: $\xi_0 = 0.75$.

Proxy

- Log likelihood $l_{\mathrm{L}}(\hat{\theta}_{\mathrm{L}})$, and $l_{\mathrm{L}}\hat{\theta}_{\mathrm{G}})$, and $l_{\mathrm{L}}(\hat{\theta}_{\mathrm{S}})$
- Left panel: GEV, Regular case (MLE exists and asymptotically normal), data from Coles (2001), center panel: TPW, MLE exsits but not asymptotically normal, data from Cheng and Amin (1983), right panel: TPW, MLE not existing, data from Cheng and Amin (1983).

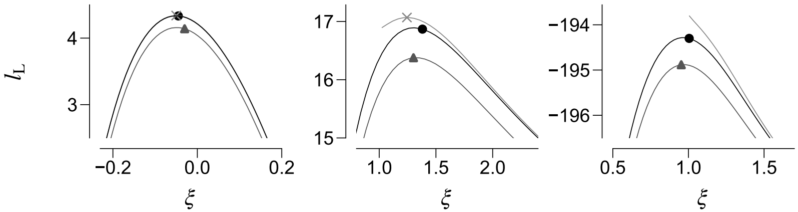

Figure: Profile likelihood, comparison of $l_{\mathrm{L}}(\hat{\theta}_{\mathrm{L}})$, $l_{\mathrm{L}}(\hat{\theta}_{\mathrm{G}})$ and $l_{\mathrm{L}}(\hat{\theta}_{\mathrm{S}})$, circle: GE, triangle: SE, cross: MLE.

Discussion

Parameter estimation of TPW and GEV

- MSE of the KLDMWR-GE, SE and CE are nearly identical.
- For the bias, KLDMWR-GE outperforms SE and CE (MLE) in most cases.
  - Exceptions are: location parameter in non-ML-existing case for TPW, location parameter in regular case for GEV.
- However, for the bias of the return levels, the superiority of GE to SE is questionable.
  - We will investigate the effect of bias correction.

KLDMWR-GE as a proxy for MLE

- For regular cases, while $\left|\hat{\theta}_{\mathrm{L}} - \hat{\theta}_{\mathrm{S}}\right| = o_p(1/\sqrt{n})$ (Anatolyev and Kosenok, 2005), we have for GE,

$$\left|\hat{\theta}_{\mathrm{L}} - \hat{\theta}_{\mathrm{G}}\right| = O(1/n),$$

- Need more investigation for non-regular cases

References

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